

# *The Null Bundle of an Einstein-Riemann Space II: Normed Null Bases*

DOMINIC G. B. EDELEN

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**1. Introduction.** This paper is a direct continuation of the results established in reference [1]. We shall refer to [1] by the Roman numeral I and cite explicit references to theorems and definitions from I by hyphenation with I. The notation is the same as that established in I and will be used without further comment. As an aid to the reader, we note here that the symbol  $E$  is used throughout to denote a given 4-dimensional, hyperbolic-normal metric space with metric tensor  $g_{AB}$ .

The purpose of this paper is to develop the theory of normed null bases and to apply the results to the problem of solving the systems of partial differential equations developed in I. This is accomplished by means of a fundamental existence theorem that is proved in Sec. 5. This theorem provides a fairly simple computational basis: all calculations can be reduced to considerations of anholonomic (scalar) fields defined over a Minkowski space. We apply these results at the end of the paper to obtain several new and interesting exact solutions of the Einstein vacuum field equations. One of the solutions is of particular interest since it represents a non-radiating gravitational field of two parallel rods in gravitational equilibrium with their mutual rotation about the common center.

**2. Representation.** Consider a system of four, covariant, linearly independent, null vector fields  $v_a^a$  ( $a = 1, 2, 3, 4$ ) on  $E$  such that  $v_a^1$  and  $v_a^2$  are complex conjugates while  $v_a^3$  and  $v_a^4$  are real. Here and throughout this paper, capital Roman letters are used for tensor indices, lower case Roman letters are used for labeling indices and for anholonomic indices, while lower case Greek letters are used exclusively for labeling indices. All indices are assumed to obey the summation convention over the range 1 through 4. The system  $v_a^a$  dual to  $v_a^a$  is defined by the requirements

$$(2.1) \quad v_a^A v_A^b = \delta_a^b \quad \text{or} \quad v_a^A v_B^a = \delta_B^A.$$

**Definition 2.1.** A set of null vector fields  $v_a^a$ , as defined above, is said to