

Trace Bending and Spherical Modifications

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Let a manifold V be the trace of a sequence of spherical modifications from M_1 to M_2 . Then V is also the trace of a sequence from $M_1 \cup M_2$ to the empty set. We consider here the problem of finding a connection between these sequences. From geometric considerations, one would expect the second sequence to arise from the first by "bending" V at some manifold intermediate between M_1 and M_2 . A general result which verifies this expectation in certain cases is given in section three. Section one contains a generalization of the h -cobordism theorem which reduces the trace bending problem to a consideration of homology. In section two, the effect of a modification on the Morse number of a manifold is computed. All manifolds are assumed to be compact, C^∞ , and *simply connected*.

The results of this paper are part of the author's Ph.D. thesis, which was written at Indiana University under the direction of A. H. Wallace.

1. A generalized h -cobordism theorem. A smooth manifold triad $(V; M_1, M_2)$ will be called simply connected if V , M_1 , and M_2 are all simply connected (see [1]). A Morse function f on a triad $(V; M_1, M_2)$ will be called ordered if, for critical points p and q ,

$$\text{index } p < \text{index } q \text{ implies } f(p) < f(q).$$

A Morse function will be called minimal if the number of critical points of each index is minimal with respect to $H_*(V, M_1)$. That is, a minimal Morse function for the triad $(V; M_1, M_2)$ has

$$b_i(V, M_1) = \text{Rk } H_i(V, M_1) + \text{Tor } H_i(V, M_1) + \text{Tor } H_{i-1}(V, M_1)$$

critical points of index i , where $\text{Rk } G$ is the rank of the group G and $\text{Tor } G$ is the minimum possible number of generators of the torsion subgroup of G .

We have the following generalization of a theorem of Smale:

Theorem 1.1. *Let $T = (V; M_1, M_2)$ be a simply connected manifold triad with $\dim V \geq 6$. Then there is a minimal ordered Morse function for T .*

Remark. (See also [5].) Taking $M_1 = M_2$ empty, we have theorem 6.1 of [2]. If M_2 is empty, we have theorem 6.5 of the same paper. On the other hand,