

*Existence and Uniqueness Theorems for a
Class of Non-Linear Singular Integral
Equations with Application to a
Hydroelastic Problem*

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Introduction. In this paper we shall be concerned with showing the existence and uniqueness of a solution, in the class of complex-valued Hölder continuous functions, of non linear singular integral equations with Cauchy type singularity. More specifically but without all hypotheses, we shall be concerned with the integral equations

$$w(z) = B(z) + \lambda C(w) + \lambda F[z, w(z), \lambda] + \frac{\lambda}{2\pi i} \int_{\Gamma} \frac{N[t, z, w(t), \lambda]}{t - z} dt,$$

where λ is a complex parameter ($|\lambda|$ small), Γ is a smooth contour in the complex t -plane, z is a point of Γ (the Cauchy principal value of the integral is understood), and for $z \in \Gamma$ and $w \in$ some closed convex set B , F and N are complex-valued functions and C is a bounded functional satisfying a certain functional smoothness condition. A definite bound for $|\lambda|$ in terms of bounds and Hölder constants for B , C , F and N and a geometrical constant for Γ will be given. We shall prove the existence and uniqueness of a solution for $|\lambda|$ not greater than this bound.

We shall prove the existence and uniqueness of a solution by successive approximations. Due to the length of the proof by successive approximations, we have proved preliminary results in the form of four lemmas.

The four preliminary lemmas mentioned are concerned briefly with: lemma 1 the inheritance of constants in Hölder conditions, lemma 2 an extension of the Plemelj–Privalov theorem with an evaluation of constants, lemma 3 an inheritance of Hölder constants for the successive approximations, lemma 4 also an inheritance statement.

There has been much interest recently in non linear integral equations as