

*On the Completeness of Cartan Connections**

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Communicated by S. S. CHERN

A Cartan connection on a principal bundle is essentially a rule which enables us to develop every smooth curve of the base space onto a smooth curve in a fibre of an associated bundle. This fibre is regarded as tangent to the base space at the initial point of the curve, and the development is a curve starting at the point of tangency.

Special cases are affine and Euclidean connections. The tangent spaces are then affine and Euclidean spaces, respectively. A torsion-free Euclidean connection is called Riemannian. Torsion of a Cartan connection will be defined below.

A Cartan connection is complete (in the sense of Ehresmann, see [1]) if every smooth curve in any of the tangent spaces, starting at the point of tangency, is the development of some curve of the base space. It is well known ([2], Theorem 4.1, p. 172) that a Riemannian connection is complete if and only if the natural metric on the base space is complete. We shall extend this result to Euclidean connections.

In general, there is no natural metric on the base space of a Cartan connection, but there is always a set of natural metrics on the total space of the principal bundle. These metrics are complete if and only if any one of them is. The main purpose of this note is to prove that a Cartan connection is complete if and only if the natural metrics on the total space of the principal bundle are complete.

Consider the following conditions on a Cartan connection ω :

- 1) ω is complete,
- 2) every regular curve of a tangent space, starting at the point of tangency, is the development of some curve of the base space,
- 3) (supposing ω to be an affine connection) every line through the origin of a tangent space is the development of a curve in the base space.

Condition 3) is called geodesic completeness.

* This research was supported by the National Science Foundation under grant NSF GP-4019.