

Some Sharp Estimations of Coefficients of Univalent Functions

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I. Let $f(z) = z + a_2z^2 + \dots$ belong to the class S , *i.e.*, let $f(z)$ be holomorphic and univalent in the unit circle K . Put $G = f(K)$ and let D denote the image of G under the transformation $w = 1/\zeta$. D is an unbounded, simply connected domain which contains the point $w = \infty$. The complement of D in the w -plane is a bounded continuum E with capacity $C(E) = 1$. The point $z = \infty$ which lies outside of G goes onto $w = 0$ under the transformation $w = 1/\zeta$ and therefore the origin 0 of the coordinate system belongs to E .

The coefficients a_n , $n = 2, 3, \dots$, of $f(z) \in S$ can be expressed as polynomials of the moments $s_k = \int w^k d\mu$, $k = 1, 2, \dots$, where μ is the natural mass distribution on E (see [2]). Further if $\tilde{\varphi}(w)$ is the function of the class Σ^{-1} (the class of functions inverse to those holomorphic, univalent and normalized in the exterior of the unit circle) which maps D onto $|z| > 1$ then

$$\log \frac{\tilde{\varphi}(w)}{w} = -\left(\frac{s_1}{w} + \frac{s_2}{2w^2} + \dots + \frac{s_n}{nw^n} + \dots\right),$$

so

$$\frac{\tilde{\varphi}'(w)}{\tilde{\varphi}(w)} = \frac{1}{w} + \frac{s_1}{w^2} + \dots + \frac{s_n}{w^{n+1}} + \dots.$$

The function $\varphi \in S^{-1}$ which is inverse to f is such that

$$\varphi(w) = \tilde{\varphi}\left(\frac{1}{w}\right)^{-1}$$

and thus

$$\frac{w\varphi'(w)}{\varphi(w)} = \frac{w\left[\tilde{\varphi}\left(\frac{1}{w}\right)^{-1}\right]'}{\tilde{\varphi}\left(\frac{1}{w}\right)^{-1}}$$