

On Locally Supersonic Plane Flows with a Weak Shock Wave

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Summary. The aim of this paper is to provide a perfect fluid model for the observed phenomena of plane transonic flow past a given profile with an attached shock wave. The starting point of the present discussion is a simple lemma on the possible hodograph configurations of the images of the front and the rear of the shock. The whole work is carried through in the hodograph plane whereas Germain, see Küchemann and Sterne (1964), first discusses general homogeneous solutions by means of a non-linear ordinary differential equation and later introduces the hodograph method to complete the discussion for plane flow. In regard to the *homogeneous* solutions the present account includes (a) the derivation of a single analytic expression which contains effectively a complete two-parameter family of shock solutions, also (b) the first comprehensive account of the non-existence, within the family of homogeneous solutions, of normal or concave shock transitions. These alone are suitable in the physical flow problems.

The homogeneous solutions in themselves being unsuitable the author considers the generalisation to a wider class of solutions with a weak shock and proves two *uniqueness* theorems for these. The first suggests that there are solutions with arbitrarily good approximations to the normal shock, and the second that in certain cases the perturbation problem for a fixed profile is well set and so the corresponding flows will be stable.

Introduction. The existence of plane transonic flows with a local supersonic region terminated by a shock wave is now well established as an experimental fact, see for example Holder (1959) (1964). Moreover, the flow regime, in particular the location of the shock waves, has been found to depend in a stable manner on the free stream Mach number. This situation has been connected by Holder with both the motion of an ideal fluid, including a weak shock discontinuity, and the interaction of the shock wave with the wake and the boundary layer. Clearly the success of these experiments has depended