

A Global Integral Representation for Abstract Harmonic Functions

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Introduction. In a previous paper [5], one of us has proved the following integral representation theorem for analytic functions on a complex analytic manifold M .

Theorem. *Let G be a relatively compact open set in M . Let A be the set of all continuous functions on \bar{G} which are analytic on G . Then there is a positive measure μ on ∂G and a Borel measurable function Q on $G \times \partial G$ such that $Q(\cdot, b)$ is analytic for each $b \in \partial G$ and for every $f \in A$ and $p \in G$*

$$(1) \quad f(p) = \int_{\partial G} Q(p, b) f(b) d\mu(b).$$

The proof depends on the fact that the analytic functions are characterized as those functions which satisfy locally equations similar to (1) using the iterated Cauchy kernel. Our object in this paper is to explicate this technique by axiomatizing the nature of the local integral representation and deducing a global integral representation. The results are immediately applicable to the harmonic functions on R^n and the parabolic functions (solutions of the heat equation) on $R^n \times R$. Moreover, they appear to cover the solutions of a considerable family of elliptic (or nearly elliptic) differential equations including harmonic and parabolic functions on general Riemannian manifolds. We hope to take up this question elsewhere.

Bauer [2] has given another abstract treatment covering harmonic and parabolic functions, but, beyond the identity of the primary models, there is little in common between the two investigations. Bauer assumes the solvability of the local Dirichlet problem and investigates the global Dirichlet problem. We are concerned only with integral representations, not with the existence of sufficiently many functions. In fact our axioms for integral kernels give no assurance that the associated class of functions contains anything besides the zero function.

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