

Differentiable Maps with Non-negative Jacobian

P. T. CHURCH¹

Communicated by G. SPRINGER

1. Introduction. In this note we observe (3) that any proper C^n map $f : M^n \rightarrow N^n$ either satisfies a global property (quasi-monotonicity) or else locally somewhere has one of two particular forms. Several related results concerning maps with non-negative Jacobian determinants are also given ((5), (6), and (13)).

Let X and Y be locally connected separable metric generalized continua [11; p. 16]. A map $f : X \rightarrow Y$ is *proper* if, for each compact set $A \subset Y$, $f^{-1}(A)$ is compact; it is *monotone* if, for each $y \in Y$, $f^{-1}(y)$ is connected; it is *light* if for each $y \in Y$, $f^{-1}(y)$ is totally disconnected. The map f is *quasi-monotone* if, for each connected open set $U \subset Y$, and component V of $f^{-1}(U)$, $f(V) = U$ [11; p. 151]. Proper open maps and proper monotone onto maps are quasi-monotone, and, conversely, any proper quasi-monotone map f can be factored uniquely $f = hg$, where $g : X \rightarrow Z$ is monotone onto and $h : Z \rightarrow Y$ is light and open [11; p. 153, (8.4)].

The differentiable proper quasi-monotone and monotone maps are studied in [1] (see also [9] and [14]), and the differentiable proper open maps are studied in [2]. In [4] Hartman and Nirenberg discuss a class of differentiable maps which, if proper are quasi-monotone [4; p. 910, Corollary 3] and thus satisfy a maximum principle [4; p. 909–910, Corollaries 1 and 2].

Examples of differentiable proper maps which are *not* quasi-monotone are given below. Let K^k and L^k be connected C^m manifolds, let $n = 1, 2, \dots$ and $k = 1, 2, \dots, n$, let $p \in L^k$, and let $g : K^k \times E^{n-k} \rightarrow L^k \times E^{n-k}$ be a C^m proper map such that $g(K^k \times \{t\}) \subset L^k \times \{t\}$ for all $t \in E^{n-k}$. (i) Let g be the product of the constant map p and the identity map, *i.e.*, $g(x, t) = (p, t)$ for all $(x, t) \in K^k \times E^{n-k}$; for $k < n$, let $L^k = S^k$. (ii) Let $k = 1$, and let the Jacobian determinant of g change sign (see (7)).

2. Conventions. Unless otherwise specified each manifold is separable and without boundary. The *branch set* B_f of a map $f : M^n \rightarrow N^n$ is the set of points

¹ N. S. F. Senior Postdoctoral Fellow.