

# *The Goursat Problem for the Partial Differential Equation $u_{xyz} = f$ : A Mirage\**

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**Abstract.** A typical boundary value problem for the hyperbolic equation  $u_{xy} = f(x, y, u, u_x, u_y)$  consists in prescribing the unknown function  $u(x, y)$  along two plane curves. This so-called Goursat problem, under suitable hypotheses on  $f$ , the curves, and the assigned functions on the curves, is known to possess a classical solution. (That is, a function  $u(x, y)$  which is continuous together with its pure mixed derivatives:  $u_x, u_y, u_{xy}$ .) However, it is shown here, by means of a simple example, that the Goursat problem for the partial differential equation  $u_{xyz} = f$  (which consists, roughly speaking, of prescribing the unknown function  $u(x, y, z)$  on three surfaces) does not, in general, possess a classical solution, even under corresponding "suitable hypotheses".

**1. Introduction.** Some interest has recently been shown (see the bibliography) in boundary value problems for the third order hyperbolic partial differential equation, in three independent variables, of Bianchi [2, 3, 4] type:

$$u_{xyz} = f(x, y, z, u, u_x, u_y, u_z, u_{xy}, u_{xz}, u_{yz}),$$

where the unknown function  $u(x, y, z)$  is never differentiated more than once with respect to each independent variable. Most of the boundary value problems for this equation in three variables are modeled after the classical boundary value problems for the second order hyperbolic partial differential equation, in two independent variables:

$$u_{xy} = f(x, y, u, u_x, u_y),$$

such as the characteristic value problem, the Cauchy problem, *etc.*

Another typical boundary value problem for the two dimensional equation is the Goursat problem, which consists, roughly speaking, of prescribing the unknown function  $u(x, y)$  along two plane curves, say,  $x = \alpha(y)$  and  $y = \beta(x)$ . If the functions  $\alpha$  and  $\beta$  are constant, then these plane curves are characteristics for the equation  $u_{xy} = f$ , and one obtains the so called "characteristic" bound-

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