

*A Complex Vectorial Formalism in General Relativity**

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Introduction. We consider a real differentiable and orientable manifold of dimension 4 (V_4) and we assume that this manifold is of class C^r ($r \geq 2$) and piecewise of class C^s ($s \geq 4$). We assume further that there exists on V_4 a field of real symmetric 2-forms of class C^{r-1} and piecewise of class C^{s-1} . In each point of V_4 , ϕ has maximum rank 4 and signature -2 .

We define on V_4 a family F_0 of tetrads by imposing the condition that in each tetrad, ϕ has the same canonical form; F_0 is also to be of class C^{r-1} , and piecewise C^{s-1} . Any other family F satisfying the same conditions can be obtained from F_0 by performing in each point a homogeneous Lorentz transformation g ; g is a function of class C^{r-1} and piecewise C^{s-1} . We shall further assume that the orientation of V_4 relative to F_0 is the same as the one relative to F and that a coherent time orientation can be defined all over the manifold. The transformations g are then restricted to the arcwise connected part of the Lorentz group.

The Riemannian connection associated to ϕ prescribes the law of parallel displacement of vectors tangent to V_4 ; given two points P_1, P_2 , a tetrad T_i ($i = 1, 2$) in each of these points, and a curve C going from P_1 to P_2 , it defines a Lorentz transformation between T_1 and T_2 . There exists thus a field of 1-form ω (connection-form) of class C^{r-2} , piecewise C^{s-2} with value in the Lie-algebra (dL) of the Lorentz group L . When one performs a Lorentz transformation, ω undergoes a linear transformation belonging to the adjoint representation of L in dL .

To each linear representation of dL corresponds an analytical description of ω . We have found convenient to represent dL as the Lie algebra of the 3-dimensional complex rotation group. The Lorentz group induces ordinary rotations in this space.

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