

Some Remarks and Examples on Continuous Systems and Moduli

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We are primarily concerned with some examples of continuous systems and moduli of general complex varieties. After preliminary remarks in Section 1, we give in paragraphs 2 and 3 examples of continuous systems whose parameter space is nowhere reduced. The first such example was given by Mumford (*Amer. J. Math.*, **85** (1962) 642–648); in our case, we compute explicitly the cohomological obstructions. In Section 4 we use the second example above to give a moduli space in higher dimensions. Paragraph 5 is devoted to some simple examples of continuous systems generated by curves in projective space; the point to be made here is that the available criterion for completeness of the characteristic system seldom applies in practice. In Appendix I we discuss the relation between continuous systems and deformations via monoidal transformations; and, in Appendix II, we discuss the Riemann–Roch theorem for prime divisors by using continuous systems.

1. Characteristic systems of continuous systems. Let X be a compact, connected complex submanifold of a complex manifold W . The notion of a *continuous system* $\{X_\tau\}_{\tau \in A}$ of compact submanifolds $X_\tau \subset W$ and for which $X = X_o$ for some $o \in A$ has been defined by Kodaira [1]. Here A is an analytic space and the X_τ are to depend holomorphically on $\tau \in A$. If $\mathbf{N}_\tau \rightarrow X_\tau$ is the normal bundle of $X_\tau \subset W$, then there is defined the *infinitesimal displacement mapping* [1]

$$(1) \quad \rho_\tau : \mathbf{T}_\tau(A) \rightarrow H^0(\mathbf{N}_\tau).$$

(If $\partial/\partial t^\alpha \in \mathbf{T}_o(A)$, then we may write

$$\rho_o\left(\frac{\partial}{\partial t^\alpha}\right) = \left.\frac{\partial X_\tau}{\partial t^\alpha}\right]_{\tau=o}$$

where $\tau = (t^1, \dots, t^m) \in A$.)

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