

A Remark on the Reduction of Unknowns in Einstein-Maxwell Field Equations in the Case of the Characteristic [(111)1]

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Introductory remark. In this paper we shall make free use of results obtained by Hlavatý in [1] and [2]. We shall also adopt the notation (with only minor changes due to press difficulties) introduced in [1] and [2] with the proviso that primed indices have the range 1, 2, 3.

The characteristic [(111)1]. The characteristic [(111)1] has been considered in Hlavatý [2]. It was shown in that paper that the metric tensor depends on six unknown scalar functions and must have the form

$$(1) \quad \begin{aligned} a) \quad g_{\lambda\mu} &= g_{a'b'} e_{\lambda}^{a'} e_{\mu}^{b'} - e_{\lambda}^{4'} e_{\mu}^{4'} \\ b) \quad g^{\lambda\nu} &= g^{a'b'} e_{a'}^{\lambda} e_{b'}^{\nu} - e_{4'}^{\lambda} e_{4'}^{\nu} \end{aligned}$$

where $g_{a'b'}$, $g^{a'b'}$ are scalars satisfying

$$(2) \quad a) \quad g_{a'b'} = g_{(a'b')}, \quad b) \quad g^{a'b'} = g^{(a'b')}, \quad c) \quad g_{a'b'} g^{a'c'} = \delta_b^{c'}$$

Besides the reduction of unknowns, a Cauchy problem was solved too. The purpose of this short paper is to show that by using the technique of the induced connections it is possible to reduce the Cauchy data by one. (For easier understanding it is advisable to read first Hlavatý [1] and [2] and then supplement it with the results brought here.)

In the following we shall use two sets of Christoffel symbols, $\Gamma_{\lambda\mu}^{\nu}$ and $\Gamma_{\lambda\mu}^{0\nu}$, belonging to two metric tensors $g_{\lambda\mu}$ and $g_{\lambda\mu}^{01}$. The metric tensor $g_{\lambda\mu}^{01}$ is defined by $g_{\lambda\mu}^{01} = \epsilon_a E_{\lambda\mu}^{a1}$ where $\epsilon_1 = \epsilon_2 = \epsilon_3 = -\epsilon_4 = 1$.

We may now state the following

Theorem 1. *The connections $\Gamma_{\lambda\mu}^{\nu}$, $\Gamma_{\lambda\mu}^{0\nu}$ are related by $\Gamma_{\lambda\mu}^{\nu} = \Gamma_{\lambda\mu}^{0\nu} + T_{\lambda\mu}^{\nu}$ where the tensor $T_{\lambda\mu}^{\nu} = T_{\lambda\mu}^{\nu}$ is given by*

$$(3) \quad 2T_{\lambda\mu}^{\nu} = 2g^{cb} I_{c1(\mu}^{a1} \nu) \partial_{\lambda)} g_{ab} - e_{\lambda}^{a1} e_{\mu}^{b1} g^{\nu\alpha} \partial_{\alpha} g_{ab} + X_{\lambda\mu}^{\nu}$$

and $X_{\lambda\mu}^{\nu}$ is a tensor not containing $\partial_{\beta} g_{a'b'}$.