

Elliptic Regularization for Symmetric Positive Systems

LEONARD SARASON

Communicated by R. PHILLIPS

Introduction. We are concerned with the identity of weak and strong solutions of boundary value problems for first order linear systems of partial differential equations in regions with irregular boundaries, and with the differentiability of such solutions when *a priori* estimates are known. In his paper on symmetric positive systems [1], Friedrichs proved 'weak equals strong' in domains with a smooth boundary, via a differentiability estimate; edges were allowed only in certain cases where the solution could be extended beyond the boundary. The writer [4] extended Friedrichs' result to include Lipschitz continuous boundaries with a sufficiently small Lipschitz constant. The idea was to approximate the problem by problems in smooth domains to which Friedrichs' result applied, and to return to the original problem with the help of a uniform differentiability estimate for solutions of the approximating problems. An inherent limitation of the latter method arises from the necessity that the approximating domains have uniformly non-characteristic boundaries. Thus the method is inapplicable if the coefficient of normal differentiation changes type across a corner, even though *a priori* differentiability estimates may exist.

In the present paper, this difficulty is overcome by use of the method of 'elliptic regularization', as suggested for this type of problem by Kohn and Nirenberg [2]. The procedure is to add to the differential operator a small multiple $-\epsilon^2 L$ of a second order elliptic operator L , together with appropriate additional boundary conditions. The new problem has a solution u_ϵ which satisfies a differentiability estimate uniformly in ϵ for small ϵ , and it follows that for some sequence of values of ϵ tending to zero, the functions u_ϵ have a weak limit u satisfying the differentiability estimate and which is a strong solution of the original boundary value problem. We shall in fact use a less sophisticated passage to the limit, for in our case solutions of the approximating problem converge strongly to a solution of the original problem as ϵ tends to 0.

Since we can construct approximating solutions which satisfy the homogeneous boundary condition exactly, the solution (in the terminology of [4]) is M -strong. (If weak and strong solutions are defined as suggested in remark 1, the solutions