

*Finite Dimensional Approximations to White Noise and Brownian Motion**

TAKEYUKI HIDA

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0. Introduction. In [2], H. Nomoto and the writer constructed a measure space (Ω, \mathbf{B}, P) , which was the projective limit of the measure space $(\Omega_n, \mathbf{B}_n, P_n)$ where Ω_n is obtained by deleting a certain $(n - 2)$ -dimensional space from the n -sphere S^n in $(n + 1)$ -dimensional Euclidean space, \mathbf{B}_n is the family of Borel subsets of Ω_n , and P_n is the restriction to \mathbf{B}_n of the uniform probability measure over S^n . In this paper we shall define a flow $\{T_t\}$ on (Ω, \mathbf{B}, P) as the limit of certain flows $\{T_t^{(2n)}\}$ on $(\Omega_{2n}, \mathbf{B}_{2n}, P_{2n})$, $n \geq 1$, which will turn out to be equivalent to the flow of Brownian motion on (Ω, \mathbf{B}, P) . Each flow $\{T_t^{(2n)}; t \text{ real}\}$ is in fact a group of rotations on S^{2n-1} .

Any one-parameter group $\{\tilde{T}_t^{(2n)}\}$ of rotations on S^{2n-1} can be expressed in suitable orthogonal coordinates in the form

$$(1) \quad \tilde{T}_t^{(2n)} = \begin{bmatrix} A_1(t) & & 0 \\ & \ddots & \\ 0 & & A_n(t) \end{bmatrix}, \quad A_k(t) = \begin{bmatrix} \cos \lambda_k t & -\sin \lambda_k t \\ \sin \lambda_k t & \cos \lambda_k t \end{bmatrix}$$

with some real λ_k . We shall consider the particular group for which the $\{\lambda_k\}$ form an arithmetical progression. Then without loss of generality we may assume that

$$(2) \quad \lambda_k = k, \quad k = 1, 2, \dots, n.$$

Using this rotation group, we define the flow $T_t^{(2n)}$ on $(\Omega_{2n}, \mathbf{B}_{2n}, P_{2n})$ as follows (for the notations see [2]):

$$(3) \quad T_t^{(2n)}(\omega_{2n}) = \begin{bmatrix} 1 & & & & 0 \\ & A_1(t) & & & \\ & & A_2(t) & & \\ & & & \ddots & \\ 0 & & & & A_n(t) \end{bmatrix} \begin{bmatrix} x_1^{(2n+1)} \\ x_2^{(2n+1)} \\ \vdots \\ x_{2n+1}^{(2n+1)} \end{bmatrix},$$

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