

# *The Near-Field Behavior of the Green's Matrix in Anisotropic Wave Motion\**

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**1. Introduction.** This paper deals with steady-state wave propagation in homogeneous anisotropic media. The Green's matrix for such a medium describes the field produced by a point-source. Mathematically, it is a fundamental solution of the field equations for the medium. The purpose of this paper is to derive the near-field behavior of the Green's matrix; *i.e.*, its behavior near the source point, for a large class of homogeneous anisotropic media. The results are applicable to most of the wave propagation phenomena of classical physics, including electromagnetic, acoustic and seismic waves in anisotropic media.

A unified discussion of the wave propagation phenomena of classical physics is possible because they are all governed by first order systems of partial differential equations of the "symmetric hyperbolic" type introduced by K. O. Friedrichs [2]. The systems can be written in the matrix form

$$(1.1) \quad ED_t u + \sum_{i=1}^n A^i D_i u = f(x, t),$$

where  $t \in R^1$  (time),  $x = (x_1, x_2, \dots, x_n) \in R^n$  (space),  $D_t = \partial/\partial t$  and  $D_i = \partial/\partial x_i$ .  $u = u(x, t) = (u_1(x, t), u_2(x, t), \dots, u_m(x, t))^*$  is a complex  $m \times 1$  (column) matrix which describes the state of the medium at position  $x$  and time  $t$ . (If  $M$  is a matrix then  $M^*$  denotes the transpose of  $M$ .)  $E$  and  $A^i$  ( $j = 1, 2, \dots, n$ ) are real  $m \times m$  matrices, characteristic of the medium, which have the properties:

$$(1.2) \quad \begin{aligned} &E \text{ and } A^i \ (j = 1, 2, \dots, n) \text{ are constant (the medium is homogeneous),} \\ &E \text{ and } A^i \ (j = 1, 2, \dots, n) \text{ are symmetric, and} \\ &E \text{ is positive definite.} \end{aligned}$$

The function  $f(x, t) = (f_1(x, t), f_2(x, t), \dots, f_m(x, t))^*$  is a complex  $m \times 1$  matrix which describes the sources acting in the medium. Some of the wave equations of classical physics are exhibited in the form (1.1) in [8].

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