

Bounds for the Eigenvalues of Nonhomogeneous Hinged Vibrating Rods

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Introduction. We consider a long, uniformly thin bar of length l which is hinged (supported) at its endpoints. Assuming that the rod has a plane of symmetry, we consider the transverse vibrations of the rod in this plane when the flexural rigidity $EI = 1$ and when the density of the rod is given by a non-negative integrable function ρ defined on $[0, l]$. The natural frequencies of the rod (see [13] p. 288 and 329) are determined by the eigenvalues of the differential system

$$(1) \quad u^{(4)} - \lambda \rho(x)u = 0, \\ u(0) = u''(0) = u(l) = u''(l) = 0.$$

We order these eigenvalues by magnitude and denote them accordingly by $\lambda_k[\rho]$ ($k = 1, 2, \dots$) where the ρ in square brackets indicates the dependence of λ_k on the density function ρ .

The main purpose of this paper is to determine lower bounds for the eigenvalues of the system (1) when ρ is restricted to a specified class of functions. We consider two such classes. The first class is the set of all functions on $[0, l]$ which have the property of being equimeasurable. Two functions are said to be *equimeasurable* if for every real number y the sets $\{f : f(x) \geq y\}$ and $\{g : g(x) \geq y\}$ have the same measure. We denote the values of this measure for a given y by $m(y)$. This determines a decreasing function m whose domain is the range of the functions in the class. Furthermore, the inverse function m^{-1} of m is uniquely determined except for at most a countable number of points on $[0, l]$. These points correspond to intervals where $m(y)$ is constant and m^{-1} may be uniquely defined by letting $m^{-1}(x) = m^{-1}(x-)$ there. We note that the function m determines a given equimeasurable class and denote this class by E_m . The symmetric function $\bar{\rho}$ with values

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