

# *Polynomials Associated with Measures in the Complex Plane\**

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**1. Introduction.** Recently an asymptotic formula was found for the lowest eigenvalues of Hankel matrices associated with measures on finite intervals of the real axis [7]. Specifically if  $\lambda_N$  denotes the lowest eigenvalue of the  $(N + 1) \times (N + 1)$  matrix  $(c_{m+n})$  ( $m, n = 0, \dots, N$ ), where

$$c_n = \int_a^b x^n d\mu(x),$$

an asymptotic formula was found for  $\lambda_N$  as  $N \rightarrow \infty$  under the assumption  $w(x)$ , the derivative of the absolutely continuous part of  $\mu$ , satisfies

$$\int_a^b \frac{\log w(x)}{(x-a)^{1/2}(b-x)^{1/2}} dx > -\infty.$$

What was crucial here was a knowledge of the asymptotic behavior, in the complex plane, of the orthogonal polynomials associated with  $\mu$ . Of course under the assumption just given this is very well known [2, Theorem 9.3]. Clearly any attempt to extend the eigenvalue result to matrices associated with measures on general sets in the plane (we shall describe these matrices below) leads to the more fundamental problem of the asymptotic behavior of the corresponding orthogonal polynomials.

Geronimus [1] considered the case of a set  $E$  which was the union of finitely many rectifiable Jordan arcs and regions whose boundaries are smooth Jordan curves, and a measure  $\mu$  on  $E$  satisfying certain conditions. He showed that in this case the monic orthogonal polynomials associated with  $\mu$  satisfy

$$\lim_{n \rightarrow \infty} \max_E |P_n(z)|^{1/n} = C(E),$$

where  $C(E)$  is the capacity of  $E$ . (See §2.) Now if the zeros of  $P_n(z)$  had no limit point in  $\Omega$ , the unbounded component of the complement of  $E$ , then

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