

Horizontal Lifts of Tensor Fields and Connections to Tangent Bundles

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Introduction. S. Sasaki [3] has studied the differential geometry of tangent bundles of Riemannian manifolds. S. Tachibana and M. Okumura [4] and P. Dombrowski [1] remarked that the tangent bundles of Riemannian manifolds or of manifolds with affine connection admit an almost complex structure which is integrable if and only if the base manifold is locally flat. A. J. Ledger and one of the present authors [2, 10] have studied linear connections induced on the tangent bundle of a manifold with linear connection.

S. Kobayashi and one of the present authors [8, 9] defined and studied prolongations of tensor fields and connections to tangent bundles. They called these prolongations the complete lifts of the geometric objects under consideration.

When a tensor field of type $(1, 1)$ satisfies a polynomial equation, its complete lift satisfies also the same polynomial equation. Thus, the complete lift of an almost complex structure is an almost complex structure and the complete lift of an f -structure, that is, a structure f of constant rank satisfying the equation $f^3 + f = 0$ is also an f -structure. The present authors [7] have recently studied the prolongation of an f -structure and also that of an almost contact structure which is a special case of an f -structure.

The main purpose of the present paper is to define prolongations of another kind, called horizontal lifts, of tensor fields and connections and to obtain the results corresponding to those obtained in [8, 9]. In Section 1, we first review the results obtained in [8, 9] for later use. In Section 2, we define the horizontal lifts of tensor fields and study properties of these lifts. In Section 3, we define and study the horizontal lifts of affine connections.

1. Vertical and complete lifts of tensor fields. We first recall, for the later use, definitions and properties of the vertical lift and the complete lift of a tensor field to tangent bundle (*c.f.* Yano and Ishihara [7]; Yano and Kobayashi [8, 9]). Let V be an n -dimensional differentiable manifold and $T(V)$ its tangent bundle (The manifold, tensor fields and affine connection under consideration are all assumed to be differentiable and of class C^∞ , and the manifold is assumed