

# *A Constructive Method for the Conformal Mapping of Domains with Corners*

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**Introduction.** The purpose of this paper is to present an effective constructive method for determining the conformal map from the interior of a rectifiable (closed) Jordan curve  $C$  onto the unit disk. The technique employed reduces the problem to solving by iteration a linear integral equation which is related to Gershgorin's equation [6].

For smooth curves the solution of Gershgorin's equation by iteration was studied by Warschawski [17] and applied by Todd-Warschawski [14]. The iterations are shown to converge uniformly with an error of order  $O(\lambda_2^{-n})$ , and an explicit error estimate is given [17]. Here  $\lambda_2$  is the least modulus of the non-trivial Fredholm eigenvalues associated with  $C$ , and it is greater than one.

If  $C$  has corners, effective methods for construction of the mapping function are known only in special cases (*e.g.*, the Theodorsen method for certain star-shaped domains—see [5, Ch. II]), and other methods are inefficient (*e.g.*, polynomial approximation—see [5, Ch. III]). Even for polygons the classical Schwarz-Christoffel formulas are of limited value since they involve constants which in general are difficult to obtain. It is therefore of interest to extend the Gershgorin method to curves with corners and to know "how good" the method is.

In Section 1 we consider curves which need not be smooth (including all polygons) and derive the integral equation of Arbenz [2], which is related to Gershgorin's equation. If  $C$  is a curve of bounded rotation without cusps, we define an iteration scheme in Section 2 which converges uniformly to the argument  $\theta(s)$  (as function of the arc length  $s$ ) of the mapping function from  $C$  onto the unit circle. The iterations are shown to converge uniformly with an error of order  $o((\lambda_2 - \epsilon)^{-n})$  for every  $\epsilon > 0$  (Theorem 1). Here again  $\lambda_2$  is the least modulus of the non-trivial Fredholm eigenvalues associated with  $C$ , and it is greater than one.

For computational purposes one is interested in an explicit estimate of the error after each iteration. The principal result (Theorem 2) gives such an estimate

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