

A Note on the Breaking of Waves on Non-Uniformly Sloping Beaches

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1. Introduction. In this note an asymptotic theory of the flow at the wave front of shallow water waves is given. The theory deals with waves which propagate into a region of undisturbed fluid over a bottom of arbitrary slope and allows to study the development of a discontinuity in the surface slope at the wave front. As an application of the theory the point is determined where the wave will break provided breaking occurs at the wave front. The result is a generalization of some formulas published earlier which refer to special shapes of the bottom. The horizontal bottom was treated by Stoker [2]. In this case there is a strict analogy between breaking and the formation of shock waves from simple waves in perfect gases. Greenspan [4] obtained the corresponding solution for waves running up beaches of constant slope. An approximate treatment of the latter example was also given later by Jeffrey [7].

Breaking may, however, occur at any point in the wave far from the wave front. Moreover breaking as it is observed, *e.g.*, on natural beaches takes place in continuous wave trains moving towards the shore and being partly reflected by it. Under these circumstances an investigation of the wave front can give but little information; one can at most expect to attain upper bounds for the distance and the time a wave travels until it breaks. It is not within the scope of this note to enter into these particulars. For a discussion of some of the questions and of the limitations of the shallow water approximation to explain breaking phenomena the reader may be referred to the excellent textbook by Stoker [1]. Attention may also be called to the interesting paper by Carrier and Greenspan [3], where an effort is made to discover general criteria for the breaking of waves.

The method used here is based on the theory of characteristics. As the differential equations of shallow water waves are of the hyperbolic type, the domain of influence of the boundary values is limited by characteristic surfaces. The front of a wave moving into a region of undisturbed fluid can therefore be affected only by the wave motion in its immediate vicinity. This is true until the wave breaks. (In simple waves the characteristic curves of one class may be each