

Real and Complex Vector Fields on Manifolds

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1. Introduction. This paper continues the study of vector fields on manifolds initiated in [23]. We will consider tangent and normal vector fields as well as complex tangent fields on almost-complex manifolds.

Let ξ be a real vector bundle over a complex X . We define the *span* of ξ to be the maximal number of linearly independent cross-sections in ξ . If X is a smooth manifold M we define *span* M to be the span of its tangent bundle. Thus *span* M is the maximal number of linearly independent tangent vector fields on M .

With any such bundle ξ we associate its Stiefel–Whitney classes $w_i \xi \in H^i(X; \mathbb{Z}_2)$. Recall that ξ is orientable iff $w_1 \xi = 0$ and that ξ is called a *spin* bundle iff $w_1 \xi = w_2 \xi = 0$.

Throughout the paper by a manifold M we will mean a closed connected smooth orientable manifold. We write $w_i M$ for the i^{th} Stiefel–Whitney class of M , and say that M is a *spin* manifold if $w_2 M = 0$.

We shall prove

Theorem 1.1. *Let n be a positive integer with $n \equiv 3 \pmod{4}$, let M be a spin manifold of dim n , and let ξ be a spin n -plane bundle over M . Then *span* $\xi \geq 3$ if, and only if,*

$$\delta w_{n-3} \xi = 0, \quad w_{n-1} \xi = 0.$$

Here δ denotes the Bockstein cohomology operator from mod 2 coefficients to integer coefficients.

Recall that Massey [10], [11] and Wu [26] have shown that if M is an n -manifold with $n \equiv 3 \pmod{4}$, then

$$\delta w_{n-3} M = 0, \quad w_{n-1} M = 0.$$

Thus as a consequence of 1.1 we have

Corollary 1.2. *If M is a spin manifold with $\dim M \equiv 3 \pmod{4}$, then *span* $M \geq 3$.*

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