

# *On a Pointwise Variational Principle for the Approximate Solution of Linear Boundary Value Problems*

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**Summary.** A method is discussed for constructing stationary functionals for the value of the solution of a linear boundary value problem, or the derivatives of such a solution, at a specified but arbitrary *point*. The procedure is applicable to any boundary value problem governed by a linearly elliptic partial differential equation, or a system of such equations, for which *global* fundamental solutions exist. The technique is particularly suited to obtain approximate results in a class of mixed boundary value problems involving a rigid body oscillating about various axes on the surface of an elastic half-space.

The procedure is illustrated by constructing stationary expressions for the solution of the Robin problem associated with the Helmholtz equation, and the class of elastodynamic problems referred to above. Method is applied to compute the displacement on the free surface of an elastic half-space due to the torsional oscillation of a finite disc situated on the surface.

**Introduction.** Most variational principles have as their stationary values integral properties of a field, usually interpretable in terms of an energy, or, in the case of problems for exterior domains, the asymptotic behavior of the field at large distances. Dirichlet's principle for Laplace's equation and the principle of minimum potential energy in elastostatics, are particular examples of the first kind, while variational principles of the Schwinger type for the amplitude of a transmitted or reflected wave are of the second kind. It is, however, also of interest to have available stationary functionals for the value of the unknown solution of a boundary value problem, or the derivatives of such a solution, at a specified but arbitrary point.

The procedure to be discussed is applicable to any linear boundary value problem governed by a linearly elliptic partial differential equation, or a system of such equations, for which global fundamental solutions can be defined, except that the eigenvalue problem and problems in which a particular lack of uniqueness prevails, are excluded. The method is also intimately connected with the ap-