

# *Note on a Stability Condition of C. T. Taam*

C. E. LANGENHOP

*Communicated by G. BIRKHOFF*

In a recent paper C. T. Taam [1] considered the equation

$$(1) \quad \frac{dX}{dt} + UX = F(t, X, \lambda),$$

where  $t$  is real,  $X$  and  $F$  take values in a Banach space  $B$  with norm  $\|\cdot\|$ , and  $U$  is a bounded linear operator in  $B$  such that for some  $\alpha > 0$ ,  $\beta > 0$  the operator  $\exp(Ut)$  satisfies the condition  $\|\exp(Ut)\| \leq \beta e^{\alpha t}$  for  $t \leq 0$ . Regarding  $F$  it was assumed, *inter alia*, that for fixed  $\rho \geq 0$  and fixed  $\lambda$  (in some parameter set), if  $\|X\| \leq \rho$  and  $\|Y\| \leq \rho$ , then

$$\|F(t, X, \lambda) - F(t, Y, \lambda)\| \leq \theta(t, \rho, \lambda) \|X - Y\|$$

for almost all  $t \in R$  (the reals) where  $\theta$  is a real-valued function of a certain type. Since we shall not be concerned directly with the dependence of  $\theta$  on  $\rho$  and  $\lambda$  we shall henceforth merely write  $\theta(t)$ . Regarding  $\theta$ , we make the following assumptions (essentially those imposed by Taam):

- $\theta$  is non-negative and measurable. Also there exists a real number  $M$
- (I) such that for all  $t \in R$ ,  $\int_t^{t+1} \theta(s) ds \leq M$ .

Using the contraction mapping principle, Taam showed that if

$$(2) \quad \sup_{t \in R} \beta \int_{-\infty}^0 e^{\alpha s} \theta(t+s) ds < r < 1$$

and

$$\sup_{t \in R} \left\| \int_{-\infty}^0 \exp(Us) F(t+s, 0, \lambda) ds \right\| < \rho(1-r)/2,$$

then (1) has a solution  $X$  such that  $\|X(t)\| \leq \rho$  for all  $t$ . Assuming also that

$$(3) \quad \limsup_{t \rightarrow +\infty} \frac{1}{t} \int_0^t \theta(s) ds < \frac{\alpha}{\beta},$$

he showed that this solution  $X$  was exponentially asymptotically stable.

Here we show that condition (3) is in fact implied by condition (2). Indeed, we prove slightly more: