

# *Classification of Space-Time Curvature Tensor IV.*

## *General Theory of Metric Classification\**

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**1. The problem to be solved.** There are two tensors in the relativistic space-time  $V_4$  which are corner stones for the classification of the curvature tensor  $R_{\omega\mu\lambda\nu}$ . One is a tensor, say  $e_{\omega\mu\lambda\nu}$ , proportional to the Levi-Civita tensor, the other is

$$g_{\omega\mu\lambda\nu} \stackrel{\text{def}}{=} g_{[\omega(\lambda}g_{\mu)\nu]}$$

where  $g_{\mu\lambda}$  is the metric tensor of  $V_4$  (its signature is supposed to be  $(+++ -)$ ). In [3] R. S. Mishra and the author based their investigation on  $g_{\omega\mu\lambda\nu}$  and found a complex frame in which the matrix of the components of the curvature tensor is

$$(1.1) \quad \left[ \begin{array}{c} \varphi \quad f \\ \bar{f} \quad \bar{\varphi} \end{array} \right] \text{Trace } \varphi = \text{Trace } \bar{\varphi},$$

where  $\varphi, f$  are  $3 \times 3$  matrices,  $\bar{f}, \bar{\varphi}$  are their complex conjugate and  $f$  is Hermitian symmetric. Because for Einstein space  $f = 0$  the matrix (1.1) is particularly handy for the Petrov classification [2] of Einstein spaces. (See section 11 in [3], where there are also some bibliographic data pertaining to the investigation of the curvature tensor.) The same authors displayed in [4] the general theory of classification with respect to the tensor  $h_{\omega\mu\lambda\nu}$  which is either  $e_{\omega\mu\lambda\nu}$  or  $g_{\omega\mu\lambda\nu}$ . Finally in [5] these authors applied this theory to the case  $h_{\omega\mu\lambda\nu} = e_{\omega\mu\lambda\nu}$  (the projective classification) and displayed all 87 general cases. In [6], [7] Hlavatý refined the concept of equivalence classes and this enabled him to find the cases disregarded in [5] (the arithmetic change, the geometric change).

When it comes to apply the theory displayed in [4] to the case  $h_{\omega\mu\lambda\nu} = g_{\omega\mu\lambda\nu}$  (the metric classification) one encounters almost insurmountable difficulties due to the fact that the basic orthogonal group preserves orthogonal components of *both tensors*  $g_{\omega\mu\lambda\nu}$  as well as  $e_{\omega\mu\lambda\nu}$ . Hence we must distinguish between null

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