

Steady Laminar Flows of an Incompressible Viscous Fluid in Curved Pipes

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Communicated by G. BIRKHOFF

Introduction. A systematic method for determining an approximate solution for steady laminar flows of an incompressible viscous fluid in curved pipes is given in this paper.

The flow in the pipe is assumed to be steady and axially symmetric with respect to the axis of the curved pipe. The two velocity components in the plane of the cross section, defined with respect to a conveniently chosen coordinate system, are expressed in terms of a stream-function. Thus, the determination of the velocity requires only the determination of the stream-function and the normal component of the velocity vector, where the normal component is perpendicular to the plane of the cross section.

It is assumed that each of these two functions can be expanded in a power series in terms of the curvature of the pipe, and shown that, by the use of these expanded forms, each of the coefficients of these series may be obtained from the solutions of associated non-homogeneous harmonic and biharmonic equations.

The details of the calculations for flow in a curved pipe of circular cross section are carried out and an expression for the rate of flow in such a pipe is obtained. It is found that the rate of flow depends on two independent variables, Reynolds number and the curvature of the pipe.

Also, the secondary flow stream-lines for a flow between two concentric curved pipes are obtained.

In calculating each additional term of the series, the terms on the right hand sides of the harmonic and biharmonic equations are found in terms of the functions determined from the solution of the preceding harmonic and biharmonic equations. Therefore, successive solutions of two main differential equations will produce as many terms as desired for the two series, dependent upon the accuracy required.

Since the determination of the terms of the series for the stream-function and the normal component of the velocity are reduced to the solutions of linear