

# *Some Positone Problems Suggested by Nonlinear Heat Generation*

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*Communciated by JÜRGEN MOSER*

**1. Introduction.** There is much current interest in boundary value problems containing positive linear differential operators and monotone functions of the dependent variable, see for example, M. A. Krasnosel'ski [1] and H. H. Schaefer [2]. We call such problems "positone" and shall examine here a particular class of them (which have been called non-linear eigenvalue problems in [2]).

One physical motivation for the problems we study concerns the temperature distribution in an object heated by the application of a uniform electric current,  $i = \lambda^{1/2}$ , (Joule heating). If the body is inhomogeneous with thermal conductivity  $K(x)$ , the electrical resistance,  $R(x, T)$ , of the object is a function of the temperature,  $T(x, t)$ , and if radiation is negligible, the resulting problem can be formulated, in some dimensionless form, as

$$(1.1) \quad \partial T / \partial t - \nabla \cdot (K(x) \nabla T) = \lambda R(x, T),$$

subject to appropriate initial and boundary conditions. In particular, we are interested in the steady states, their "stability", and their dependence upon the current,  $\lambda$ . This leads to problems of the form

$$(1.2) \quad -\nabla \cdot (K(x) \nabla T) = \lambda R(x, T),$$

subject to appropriate boundary conditions. In many cases of physical interest  $R(x, T)$  is a monotone function of  $T$  (*i.e.*, the resistance increases with temperature) and only positive solutions are of interest. In some such cases it is known that a limiting current exists beyond which positive steady states do not exist. The value of this limiting current is of great interest.

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\* Research supported by the National Science Foundation under grant No. GP-4597 at the California Institute of Technology during the academic year 1965-66. Partly supported by the U. S. Army contract DA-31-124-ARO-D-360 with New York University.

\*\* Research supported by the National Science Foundation under grant No. GP-4597 at the California Institute of Technology.