

The Problem of Hidden Variables in Quantum Mechanics

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0. Introduction. Forty years after the advent of quantum mechanics the problem of hidden variables, that is, the possibility of imbedding quantum theory into a classical theory, remains a controversial and obscure subject. Whereas to most physicists the possibility of a classical reinterpretation of quantum mechanics remains remote and perhaps irrelevant to current problems, a minority have kept the issue alive throughout this period. (See Freistadt [5] for a review of the problem and a comprehensive bibliography up to 1957.) As far as results are concerned there are on the one hand purported proofs of the non-existence of hidden variables, most notably von Neumann's proof, and on the other, various attempts to introduce hidden variables such as de Broglie [4] and Bohm [1] and [2]. One of the difficulties in evaluating these contradictory results is that no exact mathematical criterion is given to enable one to judge the degree of success of these proposals.

The main aim of this paper is to give a proof of the nonexistence of hidden variables. This requires that we give at least a precise necessary condition for their existence. This is carried out in Sections 1 and 2. The proposals in the literature for a classical reinterpretation usually introduce a phase space of hidden pure states in a manner reminiscent of statistical mechanics. The attempt is then shown to succeed in the sense that the quantum mechanical average of an observable is equal to the phase space average. However, this statistical condition does not take into account the algebraic structure of the quantum mechanical observables. A minimum such structure is given by the fact that some observables are functions of others. This structure is independent of the particular theory under consideration and should be preserved in a classical reinterpretation. That this is not provided for by the above statistical condition is easily shown by constructing a phase space in which the statistical condition is satisfied but the quantum mechanical observables become interpreted as independent random variables over the space.

The algebraic structure to be preserved is formalized in Section 2 in the