

Local C^n -operational Calculus

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1. Let X be a Banach space, and let W be a linear manifold in X . Denote by $\mathfrak{I}(W)$ the algebra of all linear transformations of X with domain W and range contained in W . For a compact interval Δ , let $C^n(\Delta)$ be the Banach algebra of all complex valued functions of class C^n on Δ ($n = 0, 1, 2, \dots$) with the norm

$$\|f\|_{n,\Delta} = \sum_{j=0}^n \sup_{\Delta} |f^{(j)}|/j!.$$

We consider an operator (= a bounded everywhere defined linear transformation of X) T with *real* spectrum $\sigma(T)$, and we fix throughout a compact interval $\Delta = [a, b]$ which contains $\sigma(T)$ in its interior. If n is a non-negative integer and W is an invariant linear manifold for T , a C^n -operational calculus for T on W is an algebra homomorphism $f \rightarrow T(f)$ of $C^n(\Delta)$ into $\mathfrak{I}(W)$ with the following properties:

- (i) $T(f) = I \mid W$ for $f(t) \equiv 1$;
- (ii) $T(f) = T \mid W$ for $f(t) \equiv t$; and
- (iii) for each $x \in W$, the mapping $f \rightarrow T(f)x$ of $C^n(\Delta)$ into X is continuous.

In (i) and (ii), I stands for the identity operator and $T \mid W$ is the restriction of T to W .

Let \mathfrak{B} denote the Borel field of the real line \mathbf{R} . If $W \subset X$ is a linear manifold, a *generalized spectral measure* on W is a mapping $E(\cdot) : \mathfrak{B} \rightarrow \mathfrak{I}(W)$ with the following properties:

- 1. $E(\mathbf{R}) = I \mid W$, and
- 2. for each $x \in W$, $E(\cdot)x$ is a regular strongly countably additive vector measure on \mathfrak{B} (cf. [3]).

Note that, for each $x \in W$, $E(\cdot)x$ is necessarily bounded (cf. [1; III. 4.5]).

Let P denote the algebra of all polynomial functions on Δ . For $n = 0, 1, \dots$, we write

$$\|x\|_n = \sup \{ \|p(T)x\|; p \in P, \|p\|_{n,\Delta} \leq 1 \}, \quad x \in W,$$

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