

# *A Strong Mixing Condition and A Central Limit Theorem on Compact Groups\**

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**1. Introduction.** A particular type of "strong mixing" implies the validity of the central limit theorem for dependent processes under appropriate further restraints [2, 6, 7]. Necessary conditions and sufficient conditions for this type of strong mixing in the case of Gaussian stationary processes have been derived [3, 4]. This paper obtains necessary and sufficient conditions for strong mixing and a related concept of uniform ergodicity in the case of a stationary process on a compact commutative group generated by taking the product of independent, identically distributed elements of the group. Sufficient conditions for strong mixing are obtained when the group is not commutative. The conditions are then used to obtain a central limit theorem on the group.

**2. Preliminaries.** Let  $\{X_k; k = 0, \pm 1, \dots\}$  be a stationary Markov process with instantaneous state space  $\Omega$  and a Borel field  $\mathfrak{B}$  of events  $B \subset \Omega$ . The one step transition probability function is

$$(1) \quad P(x, B) = P\{X_{k+1} \in B \mid X_k = x\}, \quad x \in \Omega, \quad B \in \mathfrak{B}.$$

The instantaneous invariant measure

$$(2) \quad m(B) = P(X_k \in B)$$

satisfies

$$(3) \quad \int_{\Omega} P(x, B)m(dx) = m(B) = P(B).$$

In order that all operations required be meaningful, it is required that the transition probability function be measurable ( $\mathfrak{B}$ ) in  $x$  for each  $B \in \mathfrak{B}$  and a probability measure in  $B \in \mathfrak{B}$  for each  $x \in \Omega$ . The higher step transition probability functions are given recursively by

$$(4) \quad \begin{aligned} P_{j+1}(x, B) &= P\{X_{k+j+1} \in B \mid X_k = x\} \\ &= \int P_j(y, B)P(x, dy), \quad j = 1, 2, \dots \end{aligned}$$

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