

*Support Functions of Plurisubharmonic Functions**

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Introduction. When $f(x)$ is a distribution on the real line, let $K = K(f)$ be the convex hull of its support. Clearly, K is an interval, finite or infinite. In a somewhat modernized form, Titchmarsh's convolution theorem [10] (p. 327) states that if f_1, f_2 have non-empty compact supports and $(f_1 * f_2)(x) = \int f_1(x-y)f_2(y) dy$ is the convolution of f_1 and f_2 , then $K(f_1 * f_2) = K(f_1) + K(f_2)$ where the sum stands for the set of all $x_1 + x_2$ with $x_1 \in K(f_1), x_2 \in K(f_2)$. If the supports are bounded on one side, *e.g.* from below, the equality holds for the left endpoints of the supports. If, in particular, $f_1 * f_2 = 0$, *i.e.* $K(f_1 * f_2)$ is empty, and the supports of f_1 and f_2 are bounded from one side, the theorem shows that either f_1 or f_2 has to vanish. (The first chapter of [12] contains an elementary proof of this corollary.) Lions [6] has extended Titchmarsh's theorem to several variables. All proofs published so far (see *e.g.* [2], [4], [5], [6], [7], [10]) use the Fourier-Laplace transform $\hat{f}(\xi) = \int e^{-ix\xi} f(x) dx$, which is an entire function when f has compact support. The best organized proofs are, I believe, due to Hörmander [4], whose main interest, however, is the question of singular supports. The aim of this article is to give an exposition of the subject with simple proofs. The gist of the paper is Lemma 2.2 which states that taking the least harmonic majorant of a subharmonic function is a positive linear operation and characterizes the mass of this majorant at a point on the boundary in a way reminiscent of the Phragmén–Lindelöf principle. Lemma 2.1 dealing with boundary measures is taken from [3].

Anyone interested only in a proof of Titchmarsh's theorem for one variable can limit his reading to section 2, which, apart from a proof of this result, also gives a rather broad exposition of the properties of subharmonic functions. The Titchmarsh–Lions theorem is a corollary of our Theorem 4.1, which establishes a positive linear map from a certain class of plurisubharmonic function to the class of support functions. This theorem is first proved for one complex variable (Theorem 3.1). The classical correspondence between support functions

* Dedicated to Marcel Riesz on his eightieth birthday, Nov. 16th, 1966.