

Singularities of the Reflected Riemann Matrix

A. S. DEAKIN

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1. Introduction. This is a study of the mixed initial-boundary value problem in the quarter space $x_1 \geq 0$, $t \geq 0$, where propagation is governed by the first-order hyperbolic system of equations

$$Lu = \left(E \partial/\partial t - \sum_{k=1}^N A_k \partial/\partial x_k \right) u(x, t) = 0$$

with constant matrix coefficients. The fundamental solution or Riemann matrix is a distribution or generalized function, and the initial conditions play the role of "test functions" upon which this fundamental solution acts. The Riemann matrix can be divided into two parts, one corresponding to the incident system of wavefronts and the other corresponding to the system of wavefronts formed by reflection from the hyperplane $x_1 = 0$. These we shall call the *incident Riemann matrix* and the *reflected Riemann matrix*, respectively.

In section two the incident Riemann matrix for $Lu = 0$ in the neighborhood of the boundary is obtained in terms of plane waves propagating toward the boundary. In the third section these incident plane waves are reflected from the boundary (subject to the boundary conditions) to determine the reflected system of waves. Our purpose is to find the leading term in the asymptotic estimate of the reflected Riemann matrix, which is usually the most singular part. This is done in sections five through seven, using theory of fractional integrals. The necessary lemmas as well as some of the more important properties of fractional integrals are given in section four and the appendix. The distributional nature of the asymptotic estimate is given explicitly for the three principal types of waves formed upon reflection.

When an incident wavefront impinges on the boundary and the main reflected sheets form, it may happen that one or more of the outer reflected sheets become detached from the incident sheet on the boundary (*cf.* Fig. 1.1). If this happens, branch waves form which (in mathematical terms) are generated by branch points in the reflection coefficients. The real branch waves discussed in section six are conical wavefronts linking the outer detached sheets with all inner reflected surfaces. The most singular part of the reflected Riemann matrix due to