Representability of Milnor's Functor k_{PL}.

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1. Introduction. For the introduction we shall assume that the spaces under consideration are connected and have base points, and that all maps and homotopies preserve base points. $[X, B]$ denotes the set of homotopy classes $[f]$ of maps $f : X \to B$; the associated set-valued functor is denoted by $[\ , B]$. Let $\mathcal{W}$ be the category of finite CW complexes and maps.

Let f and g be maps from a space X to Y. Recall that f and g are said to be *weakly homotopic* if $f_* = g_* : [K, X] \to [K, Y]$ for every $K \in \mathcal{W}$. (Here, $f_*[\varphi] = [f\varphi]$, $[\varphi] \in [K, X]$.) If in the definition of "H-space" we replace the term "homotopy" by "weak homotopy", we then get the definition of a *weak H-space*. One similarly defines the notion of *weak homotopy associativity, etc.*, of a multiplication μ: $X \times X \to X$ on X. If X and Y are spaces with multiplications μ and ν respectively, a map $f: X \to Y$ is called a *weak homomorphism* if $f\mu$ is weakly homotopic to $\nu(f \times f)$.

A multiplication μ on B induces a multiplication (written additively) on each $[X, B]$ by $[f] + [g] = [\mu(f, g)]$ where $(f, g): X \to B \times B$ is defined by $(f, g)(x) = (f(x), g(x))$. If $f: B \to B'$ is a weak homomorphism, then $f_* : [X, B] \to [X, B']$, $X \in \mathcal{W}$, is a homomorphism.

Milnor has developed in [5] the theory of PL-microbundles. In particular, he has defined $k_{PL}(X)$ as the set of stable equivalence classes of PL-microbundles over X. Whitney sum immediately makes $k_{PL}(X)$ an abelian monoid (it's really a group but this requires proof).

Our main result is the following.

(1.1) Theorem. *The functor k_{PL} is representable on $\mathcal{W}$ in the sense that there exists a countable CW complex B_{PL} such that k_{PL} is naturally equivalent to $[\ , B_{PL}]$. Addition on k_{PL} is represented by a weakly homotopy commutative and weakly homotopy associative H-space structure μ_{PL} on B_{PL}. Moreover, (B_{PL}, μ_{PL}) is unique in the sense that if (B, μ) is another H-space representing k_{PL} on $\mathcal{W}$ then there exists a weak homomorphism $B_{PL} \to B$ which is a weak homotopy equivalence.*

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