

# *Projection Methods in Nonlinear Numerical Functional Analysis*

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**1. Introduction.** The main purpose of this paper is to present a general and self-contained theory of the projection method for the solution of equations in a reflexive Banach space involving nonlinear operators acting in the same space. (Let us remark that with a suitable modification of our arguments the theory presented in this paper remains also valid for equations involving nonlinear operators from one reflexive Banach space into another reflexive Banach space.) Another purpose is to develop the theory in such a way as to unify the recent results on nonlinear equations obtained by Zarantonello [27], Minty [15], Browder [1, 2, 3, 4], Kachurovsky [8], Shinbrot [25], the author [19, 20] and others with the earlier results on linear equations obtained by Kravchuk [9], Mikhlin [14], Polsky [22, 23], Sobolevsky [24], Medvedev [13], Martyniuk [11, 12], Shalov [26], Lashko [10], C  a [6], Hildebrandt and Wienholtz [7], the author [17, 18, 21] and others. The projection method is used here to give a *constructive* proof of the existence and the uniqueness of solutions for a general class of nonlinear operator equations, the so-called *equations which are projectionally and strongly (weakly) solvable*.

Let us note that the class of operator equations studied in this paper is quite general. In fact, on the one hand, it includes almost all linear equations investigated in [7, 12, 13, 14, 17, 18, 22, 23, 24] for which the projection method and its actual realizations (*i.e.*, the methods of Ritz, Galerkin, and moments [9, 10, 11]) are known to be applicable as approximation methods and, on the other hand, it includes a class of nonlinear equations involving continuous, demicontinuous, and weakly continuous monotone, complex monotone,  $J$ -monotone, and other operators recently studied (from the point of view of the existence of solutions) by a number of authors [1, 3, 5, 8, 15, 19, 25, 27]. Consequently, from our two main theorems, Theorem I in Section 3 and Theorem II in Section 4, we deduce in a constructive fashion, as corollaries, some new special results as well as a number of known important results of the above mentioned authors both for linear and nonlinear equations. Furthermore, our study seems to indicate