

Central Limit Theorems for Certain Infinite Urn Schemes

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1. Introduction. We consider an urn scheme where N balls are thrown independently at a fixed *infinite* array of cells with probability p_k of hitting the k^{th} cell. Without loss of generality we may assume the cells are ordered in such a way that $p_k \geq p_{k+1}$ for $k = 1, 2, \dots$ and of course $p_k > 0$ for all k . Let

(1) $X_{N,k}$ = number of balls in the k^{th} cell after N tosses.

If the balls are thrown at the event times of a Poisson process $\{N(t); t \in [0, \infty)\}$ with parameter 1, we let

$X_{N(t),k}$ = number of balls in the k^{th} cell at time t .

(Hereafter, we reserve the notation $N(t)$ to represent a Poisson process.) The stochastic process $\{X_{N(t),k}; t \geq 0\}$ for $k = 1, 2, \dots$ is composed from mutually independent homogeneous Poisson processes with parameters p_k , $k = 1, 2, \dots$, respectively. In particular, we have

$$\Pr \{X_{N(t),k} = r\} = e^{-tp_k} \frac{(tp_k)^r}{r!}.$$

In sharp contrast, the set of random variables $\{X_{N,k}\}$ with N fixed and k varying are not mutually independent.

The purpose of this paper is to prove central limit theorems and determine the asymptotic behavior of the moments for several special functionals of the processes $\{X_{N(t),k}\}_{k=1}^{\infty}$ and $\{X_{N,k}\}_{k=1}^{\infty}$. Some explicit functionals to be considered are the random variables $Z_{N,r}$ (the number of cells containing exactly r balls after N tosses) and

(2)
$$Z_N^* = \sum_{r=1}^{\infty} Z_{N,r},$$

the number of occupied cells. (We shall use the notation $Z_{N,r}^*$ to denote the number of cells containing at least r balls.)

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