

*On Imbeddings into Orlicz Spaces and Some Applications**

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1. Introduction. We prove in this article two theorems concerning the imbedding of certain spaces defined in terms of L_p norms into certain Orlicz spaces. Specifically our results include, for domains $\Omega \subset E^n$ satisfying cone conditions:

(i) the space $W_{1,1}^1(\Omega)$ of strongly differentiable functions with derivatives in the Morrey space $L_{1,1}(\Omega)$ may be continuously imbedded in the Orlicz space $L_{\phi^*}(\Omega)$ where $\phi(t) = e^{|t|} - |t| - 1$,

(ii) the Sobolev spaces $W_p^k(\Omega)$ where $n = kp$ (which are subspaces of $W_{1,1}^1(\Omega)$) may be continuously imbedded in the Orlicz space $L_{\phi^*}(\Omega)$ where $\phi(t) = e^{|t|^{n/(n-1)}} - 1$.

Both results are shown to be sharp in a certain sense. Result (i) leads to a simplified proof of a weak form of a measure theoretic result of John and Nirenberg [6]. This weak form is that used by Moser [11], Serrin [13] and the author [18], [19] to establish Harnack inequalities for weak solutions of elliptic equations. These latter results thus do not depend on the John–Nirenberg lemma. The Harnack inequality result is used by the author [18, 19] to give alternative proofs of the Ladyzhenskaya–Ural'tseva Hölder estimates [9].

Result (ii) fills in a gap in the well known Sobolev imbedding theorems [15], and has various applications to partial differential equations. We consider here the application to eigenvalue or non-uniqueness problems for non-linear elliptic equations as studied by Berger [1, 2], Browder [3, 4], and others. The imbedding theorem (ii) permits also a gap in this work to be sealed.

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2. Preliminaries. Ω is taken to be a bounded domain in Euclidean n space, E^n .

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