

*The Monge–Ampère Equation in Magnetohydrodynamics**

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Communicated by J. B. DIAZ

1. Introduction. With the choice of the stream function and the pressure as independent variables, Martin [1], [2] has shown that the solution of problems in two-dimensional steady flow may be reduced to finding solutions of a Monge–Ampère partial differential equation. The method is based on forming proper combinations of the continuity equation and the two momentum equations so that the resultant equations may be interpreted as integrability conditions for three functions which, together with certain flow quantities, satisfy a Pfaffian system; the solution of this latter system may be made equivalent to the problem of integrating a Monge–Ampère equation, the solution of which leads to a parametric representation of the flow.

It is the purpose of the present study to show that Martin's investigation may be extended to a particular class of two-dimensional steady magnetohydrodynamic flows of a perfectly conducting fluid; namely, flows subjected to magnetic fields for which at most one component of the magnetic induction does not vanish. For this special class of flows, the governing equations simplify to a great extent, and, in fact, the three possible cases may be studied virtually simultaneously; structurally, they lead to the same differential equation, the only difference being in the definition of one of the independent variables, P , the sum of the fluid and magnetic pressures. With the choice of the stream function and the total pressure P as independent variables, it is seen that these special magnetic flows are governed by equations structurally identical to the equations considered by Martin; consequently, his analysis may be extended directly with but minor modifications.

2. The equations of motion. The equations which govern the two dimensional steady flow of an ideal, inviscid, perfectly conducting compressible fluid,

* This research was supported by a National Aeronautics and Space Administration general research grant to the University of Wisconsin.