

Uniform Asymptotic Expansions of Integrals with Many Nearby Stationary Points and Algebraic Singularities

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1. Introduction. In this paper a method will be developed to obtain the uniform asymptotic expansions of two classes of functions defined by integrals. One class of functions is of the form

$$(1.1) \quad I(k; \alpha) = \int_{\Gamma} g(t) e^{-kf(t; \alpha)} dt; \quad \alpha = (\alpha_1, \alpha_2, \dots, \alpha_p); \quad k \gg 1.$$

Here Γ is some contour in the complex t plane. The elements of α label the stationary points of f ; i.e.,

$$(1.2) \quad f_t(\alpha_\nu; \alpha) = 0, \quad \nu = 1, 2, \dots, p.$$

(The term stationary point is usually reserved for integrals where $\operatorname{Re} f = \text{const.}$ on Γ and then one uses saddle point in other cases. We shall refer to all points where f_t vanishes as stationary points.) When the elements of α are fixed (whether or not they are distinct) one could obtain an asymptotic expansion of I by (i) the method of stationary phase (when $\operatorname{Re} f = \text{const.}$ on Γ), (ii) Laplace's method (when $\operatorname{Im} f = \text{const.}$ on Γ) or (iii) the method of steepest descents [4]. One shortcoming of these methods is that if an asymptotic expansion is obtained for some fixed α and then distinct elements of α are allowed to coalesce and/or approach endpoints of Γ , this expansion will, in general, fail. Our goal is to obtain an asymptotic expansion as $k \rightarrow \infty$ which remains uniformly valid in α , even when its distinct elements are allowed to approach one another and/or boundary points. In part, the method used is based on results for the case $p = 2$ obtained in a series of papers by C. Chester, B. Friedman and F. Ursell [2], B. Friedman [8] and F. Ursell [16] and also on results of N. Levinson [10, 11, 12].

The simplest examples of I occur where $p = 1$ or 2 . In the former case, when Γ has a finite endpoint of integration one must adjust the above mentioned

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