

# *Global Solutions for Certain Systems of Quasi-Linear Hyperbolic Equations*

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**1. Introduction.** In this paper we shall be concerned with systems of conservation laws of the form

$$(1.1) \quad \begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} f(v) &= 0 \\ \frac{\partial v}{\partial t} + \frac{\partial}{\partial x} g(u) &= 0 \end{aligned}$$

where  $u$  and  $v$  are functions of  $t$  and  $x$ ,  $t \geq 0$ ,  $-\infty < x < \infty$ , and  $f$  and  $g$  are  $C^2$  functions of a single real variable. We assume that the system is hyperbolic and genuinely non-linear in the sense of Lax, [10]. We are interested in the Cauchy problem for (1.1), *i.e.*, we seek solutions of (1.1) defined in  $t \geq 0$  and also satisfying

$$(1.2) \quad (u(0, x), v(0, x)) = (u_0(x), v_0(x)), \quad -\infty < x < \infty,$$

for some given initial functions  $u_0(x)$  and  $v_0(x)$ . By a solution of this problem we mean a pair of bounded and measurable functions  $u(t, x)$ ,  $v(t, x)$  defined in  $t \geq 0$  and satisfying

$$(1.3) \quad \iint_{t \geq 0} \left[ u(t, x) \frac{\partial \phi}{\partial t} + f(v(t, x)) \frac{\partial \phi}{\partial x} \right] dx dt + \int_{t=0} u_0(x) \phi(0, x) dx = 0$$

$$(1.4) \quad \iint_{t \geq 0} \left[ v(t, x) \frac{\partial \phi}{\partial t} + g(u(t, x)) \frac{\partial \phi}{\partial x} \right] dx dt + \int_{t=0} v_0(x) \phi(0, x) dx = 0$$

for all smooth (*i.e.*,  $C^1$ ) functions  $\phi(t, x)$  having compact support.

We assume that the initial data satisfies a certain order condition, the motivation of which we get from [11]. This condition can be described as follows: If we let  $(u_i, v_i) = (u_0(x_i), v_0(x_i))$ ,  $i = 1, 2$ , for  $x_1 < x_2$ , then the Riemann

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