

Partial Regularity Results for Non-Linear Elliptic Systems

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1. Introduction. We shall be concerned with solutions $z = (z^1, \dots, z^N)$, in which each z^i belongs to the Sobolev space $H_k^{m_i}(G)$ for some $m_i \geq 1$, of the equations

$$(1.1) \quad \int_G \sum_{i=1}^N \sum_{|\alpha| \leq m_i} \zeta^i_{,\alpha} A_i^\alpha(x, Dz) dx = 0 \quad \text{for all } \zeta \in C_c^\infty(G).$$

Here $x = (x^1, \dots, x^v)$, $\alpha = (\alpha_1, \dots, \alpha_v)$ is a multi-index (each α_i is an integer ≥ 0), $|\alpha| = \alpha_1 + \dots + \alpha_v$, each m_i is an integer ≥ 1 , Dz stands for $\{z^i_{,\alpha}\}$ where $z^i_{,\alpha}$ means $D^\alpha z^i$ for $0 \leq |\alpha| \leq m_i$, and k is a real number ≥ 2 . The notations used will be those of the author's recent book [2] (see pages 4, 5) for the most part.

We shall first consider the cases where the A_i^α are of class C_μ^2 for all (x, p) and satisfy the following conditions:

$$(1.2) \quad \begin{aligned} &|A(x, p)|, |A_x(x, p)| \leq M V^{k-1}, \quad |A_p|, |A_{px}|, |A_{pp}| \leq M V^{k-2}, \\ &\sum_{i,j=1}^N \sum_{|\alpha|=m_i} \sum_{|\beta|=m_j} A_{i p \beta^i}^\alpha(x, p) \pi_\alpha^i \pi_\beta^j \geq m^* V^{k-2} |\pi|^2, \quad m^* > 0, \\ &V^2 = 1 + \sum_i \sum_{|\alpha| \leq m_i} (z^i_{,\alpha})^2, \quad k \geq 2, \quad (p = \{p_\alpha^i\}, 0 \leq |\alpha| \leq m_i), \end{aligned}$$

where

$$(1.3) \quad |A_p|^2 \text{ means } \sum_{i,j=1}^N \sum_{|\alpha| \leq m_i} \sum_{|\beta| \leq m_j} (A_{\alpha p \beta^i}^i)^2, \quad \text{etc.}, \quad m^* > 0.$$

We also consider the solutions of equations (1.1) in the case where all the $m_i = 1$, in which case we write the equations in the form

$$(1.4) \quad \int_G [\zeta^i_{,\alpha} A_i^\alpha(x, z, \nabla z) + \zeta^i B_i(x, z, \nabla z)] dx = 0 \quad \text{for all } \zeta \in C_c^\infty(G),$$

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