

On the Existence, Uniqueness, and Stability of Solutions of the Equation $\sigma'(u_x)u_{xx} + \lambda u_{xtx} = \rho_0 u_{tt}$

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1. Introduction. This paper presents a study of the mixed boundary-initial value problem for the third-order partial differential equation

$$(E') \quad \rho_0 u_{tt} = \sigma'(u_x)u_{xx} + \lambda u_{xtx}, \quad (x, t) \in (0, 1) \times (0, \infty),$$

where λ and ρ_0 are positive constants. (In the future (\cdot) will denote differentiation with respect to u_x . Arguments of functions, if not stated explicitly, will be x and t .) It will be shown that if $\sigma'(\xi) > 0$ the problem is well set in the sense that there exists a unique solution which is stable with respect to perturbations in the initial data. Moreover, it will be shown that the solution decays to zero as t tends to infinity.

A physical prototype of the problem studied here arises when one considers purely longitudinal motions of a homogeneous bar of uniform cross-section and unit length. If we denote by x the position of a cross-section (which is assumed to move as a vertical plane section) in the homogeneous rest configuration of the bar, by $u(x, t)$ the displacement at time t of the section from its rest position, by $\tau(x, t)$ the stress on the section at time t , and by $\rho_0 > 0$ the constant density of points in the rest position, then the equation of motion becomes

$$(1.1) \quad \rho_0 u_{tt} = \tau_x, \quad (x, t) \in (0, 1) \times (0, \infty).$$

If one takes the ends of the bar (in our case the points 0 and 1) to be clamped for all times $t > 0$, then the displacement u must satisfy the auxiliary conditions

$$(A) \quad u(0, t) = u(1, t) = 0, \quad t > 0.$$

One may now obtain a differential equation for the displacement u by making a specific assumption about the dependence of the stress τ on the displacement. The simplest such assumption is Hooke's law which asserts that

$$(1.2) \quad \tau = E_0 u_x,$$