

# *Asymptotic Results for a Class of Integral Operators Over Groups*

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**0. Introduction and notation.** Let  $\Gamma$  be a locally compact topological group and let  $| \cdot |$  be a fixed left invariant Haar measure on  $\Gamma$ . Then if  $F$  is a suitably restricted measurable function on  $\Gamma$  and  $V$  is a Borel set of  $\Gamma$  of finite non-zero measure, we define the Finite Toeplitz Operator  $F_V$  on  $L^2(V)$  by

$$(F_V \varphi)(\gamma) = \int_V F(\tau^{-1}\gamma)\varphi(\tau) d\tau$$

Furthermore, we define  $W_\Gamma$  to be the set of all sequences  $\{V_n\}$  of Borel sets of  $\Gamma$  of finite non-zero measure satisfying

$$\lim_{n \rightarrow \infty} \frac{|\gamma V_n \Delta V_n|}{|V_n|} = 0 \quad \text{for all } \gamma \in \Gamma.$$

The prototype of the asymptotic theorems we obtain is the following: Let  $\Gamma$  be Abelian with dual group  $G = \hat{\Gamma}$ ,  $F \in L^1(\Gamma) \cap A(\Gamma)$  and Hermitian ( $F(-x) = \overline{F(x)}$ ),  $\{V_n\} \in W_\Gamma$ , and let  $\{\lambda_{n,j}\}$  denote the sequence of eigenvalues of  $F_{V_n}$ . Then if  $F = \hat{f}$  where  $f \in L^1(G)$  and Haar measures are normalized on  $\Gamma$  and  $G$  by Fourier Inversion,

$$(*) \quad \lim_{n \rightarrow \infty} \frac{\text{number of } \lambda_{n,j} \in [a, b]}{|V_n|} = |\{x \in G : a \leq f(x) \leq b\}|,$$

if  $0 \notin [a, b]$  and  $|\{x \in G : f(x) = a \text{ or } f(x) = b\}| = 0$ . This generalizes the classical result of Szego ([1]) as well as the results of Krieger ([2]) which treat the case of compactly generated Abelian  $\Gamma$ . Conversely, if  $\{V_n\}$  is a sequence of Borel sets of  $\Gamma$  of finite non-zero measure for which (\*) holds for *all* such  $F$  we then show that  $\{V_n\} \in W_\Gamma$ .

This asymptotic distribution theorem (\*) is derived from an asymptotic formula for the distribution of the eigenvalues of Finite Toeplitz Operators in general *unimodular* groups; in the Abelian case the result takes a particularly nice form due to the availability of dual groups.

We then extend our considerations to more general operators over Abelian