

*Optimum Heat Transfer and Network Programming**

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0. Abstract. Of concern is a network in which the conductance of certain branches is variable. The problem posed is the maximization of the input conductance of the network under the constraint that the sum of the branch conductances has a fixed bound. It is shown that at the optimum state the conductance of the variable branches should be proportional to the current through them. This property leads to inequalities which serve to give a numerical estimate of the maximum input conductance.

1. Introduction. A general problem of heat transfer is the design of machinery so that the structure can carry away excess heat without exceeding weight limits. For example cooling fins are used on the cylinders of air cooled engines and the design problem is to determine the optimum shape of the fins. This problem was studied in two previous papers [2] and [7]. In this paper the machine is treated as a lumped network having a finite number of conducting branches. Certain branches are allowed to vary but the total weight must not exceed a given limit. The problem is to maximize the joint conductance Γ of the network subject to the constraints.

The resulting mathematical problem may be characterized as a maximizing problem of non-linear programming. This suggests that there is a dual minimizing problem. Following such a line of investigation leads to inequalities giving upper and lower bounds for the joint conductance Γ . These bounding inequalities are suitable for giving an estimate of Γ .

A continuous system may be regarded as a limiting case of a lumped network and there should be analogies between the properties of the continuous systems and the properties of the discrete systems. For example it was shown in reference [2] and [7] that the optimum cooling fin has the property that the magnitude of the temperature gradient is a constant. In this paper an analogous property is found to hold for the optimum networks. The analogy is that the temperature difference across the variable branches is constant.

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