

*Optimum Shape of a Cooling Fin on a Convex Cylinder**

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0. Abstract. The problem considered is that of maximizing the heat dissipation of a cooling fin of fixed weight attached to a cylinder with a convex cross-section by properly tapering the fin. It is assumed that Newton's law of cooling holds and that the boundary of the cylinder has a constant temperature. In a previous paper R. J. Duffin (*A Variational Problem Relating to Cooling Fins*, J. Math. Mech. **8** (1959) 47–56) considered the special case of cylinders of circular cross-section and proved that for the optimum taper the temperature gradient is a constant. Our method is to convert the differential equation for the heat flow into a saddle point variational problem. The solution of this variational problem shows that for the optimum taper the temperature gradient vector again has constant magnitude. This criterion leads to explicit formulae for the thickness of the fin at each point.

1. Introduction. Cooling fins are used to conduct heat away from machines to the ambient medium. Common examples are the fins on the cylinders of air cooled internal combustion engines. It is not difficult to see that in order to economize on weight the fin should taper, narrowing in the direction of heat flow. This gives rise to a definite mathematical problem which may be phrased in this way—*how should the thickness of the fin be tapered so as to minimize the weight of the fin for a given rate of dissipation of heat.* This will be called Problem O.

In a previous paper [1] one of us gave an exact solution of this problem for the case of a disk fin on a circular cylinder assuming Newton's law of cooling. The solution proved to be surprisingly simple; the thickness of the fin is tapered so as to satisfy the following criterion—*the temperature should be a linear function of the distance along a radius.* This criterion had previously been proposed by E. Schmidt [2] but without convincing proof.

The present paper concerns a cooling fin on a convex cylinder. The two dimensionality complicates the problem because the equation of heat conduction is now a partial differential equation instead of an ordinary differential equation.

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