

Representability of Products in a Cohomology Theory

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1. Introduction. A common way of manufacturing (generalized reduced) cohomology theories h^* on the category \mathcal{C} of finite CW complexes with base points and base point preserving (continuous) maps is by using a spectrum \mathbf{A} . An important result due to E. H. Brown [1, Thm. II] asserts that if a given theory has countable coefficients then it arises in just this manner from an Ω -spectrum whose spaces can be selected to be countable CW complexes. In [6] G. W. Whitehead introduces the notion of a pairing of spectra which, in turn, leads to a product structure on the associated cohomology h^* . It's natural to ask if the converse is true in the sense that every product on h^* arises from a pairing of spectra. The main results of Part II, Theorems (6.3) and (7.7), assert that the answer to this question is yes, provided we weaken Whitehead's definition to that of a "weak pairing."

By calculating the set of pairings of Eilenberg–MacLane spectra we obtain as an application the existence and uniqueness of cup products in ordinary cohomology, Corollary (9.3). The original proof of uniqueness, due to Steenrod, was to have appeared in Vol. 2 of Eilenberg and Steenrod's *Foundations* but at present no proof yet seems to be in the literature.

Our paper is organized into three parts. Part I is concerned with products on representable half exact functors. Section 2 introduces the notion of a pairing of contravariant functors and states the main results of Part I, Theorems (2.1) and (2.2), the proofs of which occupy Sections 4 and 5. In Section 3 we examine the relations between internal and external products on a half exact functor showing that they are in one-one correspondence.

Part II deals with cohomology theories. In Section 6 we first define the notions of a pairing of cohomology theories and of a weak pairing of spectra, and then state our main result of Part II, Theorem (6.3). The next section examines axioms concerning products on a cohomology theory and their relationship to axioms

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