

The Invariance of Wave Operators Associated with Perturbations of $-\Delta$

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1. Introduction. Let A_0 denote the operator $-\Delta$ in $L^2(\mathbb{R}^n)$ ($n \geq 2$); let A_1 be the operator $-\Delta$ with a Dirichlet or Robin boundary condition in $L^2(\Omega)$, where Ω is the domain exterior to a C^2 , compact hypersurface in \mathbb{R}^n ; and let A_2 be the operator $-\Delta + q(x)$ in $L^2(\mathbb{R}^3)$. We assume that the potential $q(x)$ is an L^2 function which is locally Hölder continuous except for a finite number of singularities and which for large x satisfies

$$(1.1) \quad q(x) = O(|x|^{-5/2-h})$$

for some $h > 0$.

A_1 and A_2 have (improper) eigenfunction expansions provided by corresponding sets of distorted plane waves. In this paper we will use these eigenfunction expansions to give new proofs of the existence, completeness, and invariance of the Schroedinger wave operators $W_{\pm}(\beta(A_j), \beta(A_0))$ ($j = 1, 2$) for real-valued functions β which are piecewise strictly monotone.

The wave operators relating the free-space wave equation with the wave equation

$$(1.2) \quad u_{,tt} = -A_j u$$

($j = 1, 2$) are unitarily equivalent to the Schroedinger wave operators for

$$\{A_j^{1/2} \oplus (-A_j^{1/2}), A_0^{1/2} \oplus (-A_0^{1/2})\}$$

(see [7]). Consequently, the existence of the wave operators for (1.2) and their relationship with the eigenfunction expansions, established in [11] and [12] for $j = 1$ and by Thoe [14] for $j = 2$, also follow from the results of this paper (except that Thoe requires only $q(x) = O(|x|^{-2-h})$).

In a short appendix we will indicate how eigenfunction expansions can be

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