

*Measures Orthogonal to Polydisc Algebras**

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1. Introduction. Complex Borel measures λ on the unit circle T whose Fourier coefficients

$$\hat{\lambda}(n) = \int e^{-in\theta} d\lambda$$

vanish for $n \leq 0$ have been studied a good deal. One possible reason for this, though I believe not the historical one, is that they are the measures that annihilate the algebra $A(T)$ of complex continuous functions f on T whose Fourier coefficients

$$\hat{f}(n) = \frac{1}{2\pi} \int_0^{2\pi} e^{-in\theta} f(e^{i\theta}) d\theta$$

vanish for $n < 0$. $A(T)$ is closely related to holomorphic functions on the open unit disc D , for it is the restriction to T of the algebra of functions that are continuous on the closed unit disc \bar{D} and holomorphic on its interior D . Possibly the first, and certainly the most famous fact about the measures that annihilate $A(T)$ is the F. and M. Riesz theorems [6], which say that a nontrivial annihilating measure and linear measure on T have the same null sets. This in turn implies, by Bishop's general Rudin–Carleson theorem [1], the Rudin–Carleson theorem: if E is a closed subset of T with linear measure zero, then every continuous function on E is the restriction to E of a function in $A(T)$.

One generalization of $A(T)$ is the algebra $A(T^N)$ ($N \geq 2$) of complex continuous functions f on the N -torus T^N whose Fourier coefficients

$$\hat{f}(n) = \left(\frac{1}{2\pi}\right)^N \int_0^{2\pi} \cdots \int_0^{2\pi} \exp(-i\langle n | x \rangle) f(e^{ix_1}, \dots, e^{ix_N}) dx_1 \cdots dx_N$$

vanish whenever some component of n is < 0 . Here n belongs to Z^N where Z is the integer group; and $\langle n | x \rangle$ is the inner product of n with x :

$$\langle n | x \rangle = n_1 x_1 + \cdots + n_N x_N.$$

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