Exotic Cobordism Theories Associated with Classical Groups

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§1. Introduction. Among the standard notions of cobordism theory is the concept of 'cobordism of manifolds with additional structure on the normal bundle'. Briefly, one begins with a fibration

$$f : B \to BO$$

where BO is the classifying space for the stable orthogonal group O, and a (B, f) manifold is a compact differentiable manifold M^n together with a homotopy class of liftings of the normal map of M, $\nu : M \to BO$, to B. The cobordism group of closed n-dimensional (B, f) manifolds will be denoted $\Omega_n(B, f)$.

Let $f_r : B_r \to BO_r$ be the fibration induced from f by the inclusion of BO_r in BO and let $g_r : B_r \to B_{r+1}$ be the fibre map induced by the inclusion of BO_r in BO_{r+1}. The Thom space $Tf_r^*(\gamma^r)$ is formed by collapsing to a point, ∞, all vectors of length at least one in the bundle $f_r^*(\gamma^r)$ over B_r induced from the canonical r-plane bundle γ^r over BO_r. The fibre maps g_r induce maps

$$Tg_r : \Sigma Tf_r^*(\gamma^r) \to Tf_{r+1}^*(\gamma^{r+1})$$

where Σ denotes the suspension, defining a spectrum $\mathbf{TB} = \{Tf_r^*(\gamma^r), Tg_r\}$.

The primary result in this form of cobordism theory is the generalized Pontrjagin–Thom theorem:

The cobordism group of n-dimensional closed (B, f) manifolds is isomorphic to the n-th stable homotopy group of the spectrum **TB**; i.e.

$$\Omega_n(B, f) \cong \lim_{r\to\infty} \pi_{n+r}\cdot(Tf_r^*(\gamma^r), \infty).$$

Note: A full discussion of the above ideas may be found in Lashof [3].

One way to construct such cobordism theories is to be given a representation $\rho : G \to O$, G being a topological group. One then has induced a fibration on the classifying space level

$$B\rho : BG \to BO.$$

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