

# Fiber Bundles with Singularities

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**1. Introduction.** Let  $M$  and  $N$  be connected topological spaces. A *singular fibering* is a map  $f: (M, A_f) \rightarrow (N, f(A_f))$  such that

- (1)  $A_f \neq M$  and is closed;
- (2)  $f|A_f$  is a homeomorphism and  $f^{-1}(f(A_f)) = A_f$ ;
- (3) for each component  $W$  of  $N - f(A_f)$ ,  $f|f^{-1}(W)$  is the projection map of a fiber bundle with base space  $W$  (and possibly empty fiber). Moreover, we may suppose that
- (4)  $A_f$  is a minimal set satisfying (1), (2), and (3).

This concept is a generalization of one introduced by Montgomery and Samelson [20], and related versions have been studied by Conner and Dyer [11], Mahowald [19], and Antonelli [1]. Other concepts of singular fibering have been investigated in [3], [15], [22], [26], [31], [32], and [33]. For a description of the relation between [32] and this paper, see (1.8).

Given maps  $\psi: P \rightarrow Q$  and  $\omega: R \rightarrow S$ , define  $\psi \times \omega: P \times R \rightarrow Q \times S$  by  $\psi \times \omega(p, r) = (\psi(p), \omega(r))$ . Define the open *cone*  $c(P)$  as the identification space obtained from  $P \times [0, 1)$  by identifying  $P \times \{0\}$  to a point  $p^*$ , let  $\iota$  be the identify map on  $[0, 1)$ , and let the *cone map*  $c(\psi): c(P) \rightarrow c(Q)$  be the map induced by  $\psi \times \iota$ . Let  $\iota_k$  be the identity map on  $E^k$ .

**1.1. Examples.** In each of (i), (ii), and (iii) below a map  $\psi$  is defined, and  $c(\psi) \times \iota_k$  is a singular fibering with singular set  $A_f$ , a  $k$ -cell ( $k = 0, 1, \dots$ ).

(i) Let  $m = 2, 4$ , or  $8$ , and let  $\psi: S^{2m-1} \rightarrow S^m$  be a locally trivial fiber map with fiber  $S^{m-1}$  (or a homotopy 3-sphere in case  $m = 4$ ); examples are the Hopf maps [28; p. 105–110].

(ii) Let  $T^m$  be a homotopy sphere such that  $c(T^m)$  is an open  $(m + 1)$ -manifold, and let  $\psi: T^m \rightarrow E^0$  ( $m = 0, 1, \dots$ ).

(iii) Let  $\psi: S^1 \rightarrow S^1$  be a  $d$ -to-1 covering map with  $d > 1$ , e.g.,  $\psi(\theta) = d \cdot \theta \pmod{2\pi}$ .

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