

Dynamics of a Spinning Gas Cloud

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Abstract. A class of exact solutions of the inviscid hydrodynamical equations is found, representing motions of a cloud of compressible gas expanding freely in an otherwise empty space. These solutions have a symmetry-group $O(4)$ whose six generators are constants of the motion. The generators are linear combinations of components of angular momentum and of vorticity. The same symmetry occurs in the dynamics of the classical ellipsoids of Dirichlet, Dedekind and Riemann.

I. Introduction. 107 years ago there appeared three papers, under the names of Dirichlet [1], Dedekind [2] and Riemann [3], discussing the dynamics of an incompressible fluid ellipsoid held together by gravitational forces. The Dedekind and Riemann ellipsoids were then forgotten for a hundred years until Chandrasekhar [4] and Lebovitz [5] revived them. As Chandrasekhar stated in a recent historical review [6], "the reason for this total neglect must, in part, be attributed to a spectacular discovery by Poincaré which channeled all subsequent investigations along directions which appeared rich with possibilities. . . . The grand mental panorama that was thus created was so intoxicating that those who followed Poincaré were not to recover from its pursuit." However, the example of Poincaré can hardly be considered a sufficient excuse for consigning the Riemann ellipsoids to oblivion. After all, they occupy 30 pages of the single volume of Riemann's collected works [7], in which 55 pages are devoted to Riemann surfaces, 39 to Riemann integration, 20 to the Riemann invariants of compressible fluid dynamics, 16 to Riemannian geometry, and 9 to the Riemann hypothesis [8].

A central feature in the work of Dirichlet, Dedekind and Riemann is a principle of duality, first formulated explicitly by Dedekind [2]. The principle associates with every ellipsoid an adjoint ellipsoid having the same size and shape. The motions are related in such a way that the angular momentum of the original ellipsoid is equal to the vorticity of the adjoint and vice versa. The Dedekind ellipsoids, which have a fixed orientation in space, are the adjoints of the previously known Jacobi ellipsoids, which rotate without any internal motion. Riemann [3] determined the most general configuration of a spinning ellipsoid,