

A Characterization of Analyticity. III

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Communicated by E. R. LORCH

1. Introduction. This paper continues the development of the theory of families of *real* analytic functions characterized by Lipschitz type conditions initiated in [4] and continued in [6]. Handled here is the case of solution spaces of *elliptic* homogeneous constant coefficient partial differential equations. It will be shown that a suitably defined family of functions on a Euclidean space E into the reals R , either satisfies a uniform Lipschitz condition and consists of analytic functions, or contains arbitrary continuous functions satisfying no Lipschitz condition whatsoever.

2. Notation and definitions. Let R denote the real numbers and ω the positive integers. Throughout this paper E will denote a fixed Euclidean space of dimension p , $p \in \omega$. For $x \in E$, $\delta > 0$, set $U_x(\delta) = \{y \in E; \|y - x\| < \delta\}$, $B_x(\delta) = \{y \in E; \|y - x\| = \delta\}$, and set $U(\delta) = U_0(\delta)$, $B(\delta) = B_0(\delta)$, $U = (U1)$ and $V = B(1)$. Let G denote the rotation group (group of unitary linear transformations of E into E) of E . Let $F(E, R)$ be the family of continuous functions on open subsets of E into R . For $f \in F(E, R)$, and A an invertible map of E onto E , we set $(fA)(x) = f(A(x))$ for all $x \in \{t \in E; A(t) \in \text{dom } f\}$.

Let $F \subseteq F(E, R)$. Then F is called a T family if

- (1) For $f, g \in F$, F contains the function h such that $h(x) = f(x) + g(x)$ for $x \in \text{dom } f \cap \text{dom } g$.
- (2) For $f \in F$, $r \in R$, $rf \in F$.
- (3) For $f \in F$, S an open subset of $\text{dom } f$, the restriction $f|_S$ of f to S lies in F .
- (4) For $f \in F$, $x \in E$, F contains the translate fA_x , where $A_x(y) = x - y$ for $y \in E$.

F is called an R family if for $f \in F$, $r > 0$, F contains the function fA_r , where $A_r(x) = rx$ for $x \in E$. F is called a G family if for $f \in F$, $g \in G$, $fg \in F$. F is called an L family if there exists $N > 0$, such that for $f \in F$, $\bar{U} \subseteq \text{dom } f$, we have

$$|f(x) - f(0)| \leq N \sup \{|f(t)|; t \in U\} \|x\|$$

¹ This research supported by NSF Grant GP-6311.